

Written Exam for the B.Sc. / M.Sc. in Economics 2010-I

**Corporate Finance and Incentives**

Elective Course/ Master's Course

February 18, 2010

(4-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub questions that do not necessarily have equal weight.

Please provide intermediate calculations.

### **Problem 1 (Fixed Income, 25%)**

Assume a bond market consisting of 3 risk free bonds, all with annual coupon payments and a face value of DKK 100. A 1 year 2% bullet bond priced at 101, a 2 year 4% serial bond priced at 104 and a 3 year 4% annuity priced at 104.5. The annuity has annual cash flows of 36.035% of face value.

- 1) Compute the annual total cash flow payments for each bond.
- 2) Calculate the corresponding 1, 2 and 3 year zero-coupon bond discount rates and zero-coupon bond yields. Calculate the 1 year zero-coupon forward rates.

Hint: we can express the entire bond market as an equation system  $\pi = Cd$ , where  $\pi$  is a bond price vector,  $C$  is the cash flow matrix and  $d$  is the zero-coupon bond discount rate.

- 3) How is Macaulay duration defined and what are the two interpretations of the duration measure?

Assume a flat term structure and a yield to maturity on the 3 year annuity bond of 2.50%.

- 4) Calculate the Macaulay duration and the modified duration on the 3 year annuity bond.
- 5) Assume a 125 basis point parallel downward shift in the yield curve. What is the approximate %-change in the price of the 3 year annuity bond as estimated by the modified duration measure?
- 6) Calculate the exact %-change in the annuity's price. What causes our estimation error from using modified duration in question 5?

## Problem 2 (Mean Variance and CAPM, 25%)

As an investment manager with a long only mandate (i.e. no shorting is allowed), your focus is on two asset classes: European Equities and Emerging Market Equities have the following historical returns:

Table 1: 5 year historical returns.

Year	European Equities*	Emerging Market Equities*
2008	-38.9%	-45.9%
2007	6.0%	33.2%
2006	19.1%	28.5%
2005	24.9%	35.3%
2004	12.2%	16.1%

\* European Equity returns are measured by the Morgan Stanley MSCI Europe Total Return Index and Emerging Market Equity returns by the Morgan Stanley MSCI Emerging Market Free Total Return Index. Both indices are measured in local currencies.

- 1) What is the expected return, variance and standard deviation on the two asset classes? (NOTE! Do not correct for degrees of freedom. For instance find  $\sigma^2$  rather than  $s^2$ )
- 2) Calculate the covariance and correlation between the two asset class returns and list the covariance matrix.
- 3) Assume that you create a portfolio that consists of 20% European Equities and 80% Emerging Market Equities. What is the expected return, variance and standard deviation of the portfolio?
- 4) What is required of the correlation between any two assets in a long only portfolio in order to have even the slightest systematic risk diversification benefits? What is required of the correlation in order to eliminate all systematic risk and what return should be expected of a zero risk portfolio assuming no arbitrage?

Assume that the CAPM model holds and that the risk free rate equals 1.5%, the expected return on the market portfolio equals 6.1% and it has a standard deviation of 25%. Assume further that Emerging Market Equities have a covariance of returns with the market portfolio of 0.08.

- 5) What is the beta of Emerging Market Equities? What kind of risk-sensitivity does beta measure? What is the interpretation of the riskiness of Emerging Market Equities based on the found beta?
- 6) What is the expected return on Emerging Market Equities according to the CAPM model?

Assume that the index value on Emerging Market Equities today is USD 27. The consensus among a large group of analysts is that holding the index will pay dividends of USD 0.2 in one year and that the ex-dividend index value will be 29 one year from now.

- 7) Say the CAPM model's prediction about the expected return on Emerging Market Equities is accurate. Do the analysts then over- or undervalue Emerging Market Equities? Why?

**Problem 3 (Options, 20%)**

Consider an economy where a share in Candyman & Co. is trading at €77 and the yearly continuous compounded risk-free rate is 7%. A zero-cost forward on Candyman & Co. is traded in the market. The forward is settled in one year.

- 1) Estimate the zero-cost forward price of the forward (hint: use continuous compounding)

Assume a European call option on Candyman & Co. is traded in the market and that the the share price of Candyman & Co. satisfies the Black-Scholes assumptions. The option expires in two years and has a strike price of €85 and the volatility of the stock's log return is 10%.

- 2) Estimate the price of the European call option.
- 3) Estimate the price of a comparable European put option.
- 4) Comment briefly on how the price of the European call would be related to the price of the European put if their strike price was similar to the zero-cost forward price in question 1.
- 5) Comment on the possibility of arbitrage if another European call option with identical expiration is trading with another implied volatility.

Now assume that the Candyman & Co. decides to pay a dividend of €10 such that the ex-dividend date is just prior to expiration of the European call option.

- 6) Estimate the price of the European call option when Candyman & Co. pays a dividend

**Problem 4 (Essay questions, 30%)**

- 1) Define the concept of real options and explain how they add value to an investment
- 2) Explain whether there are tax advantages to leasing
- 3) Discuss the differences between direct and indirect bankruptcy costs.

## Fixed Income

$$\pi = Cd$$

$$y(0, t) = \left(\frac{1}{d_t}\right)^{\frac{1}{t}} - 1 = r_t$$

$$\text{yield to maturity, } y \text{ solves: } \pi = \sum_{t=1}^T \frac{c_t}{(1+y)^t}$$

	$c_t$	$i_t$	$\delta_t$
	payment	interest	deduction of principal
Annuity	$F\alpha_{\tau R}^{-1}$	$R\frac{F}{\alpha_{\tau R}}\alpha_{\tau-t+1 R}$	$\frac{F}{\alpha_{\tau R}}(1 - R\alpha_{\tau-t+1 R})$
Bullet	$RF$ for $t < \tau$ $(1+R)F$ for $t = \tau$	$RF$	0 for $t < \tau$ $F$ for $t = \tau$
Serial	$\frac{F}{\tau} + R(F - \frac{t-1}{\tau}F)$	$R(F - \frac{t-1}{\tau}F)$	$\frac{F}{\tau}$

$$D(c; r) = \sum_{t=1}^T tw_t$$

$$K(c; r) = \sum_{t=1}^T t^2 w_t$$

$$w_t = \frac{c_t}{(1+r)^t} \frac{1}{PV(c; r)}$$

## Mean-Variance Optimization, CAPM, APT and Factor Models

$$E(\tilde{r}_i) = \sum_{s=1}^S q_s \times \tilde{r}_{i,s} = \bar{r}_i$$

$$Var(\tilde{r}_i) = E[(\tilde{r}_i - E(\tilde{r}_i))^2] = \sum_{s=1}^S q_s \times (\tilde{r}_{i,s} - \bar{r}_i)^2 = \sigma_i^2$$

$$Cov(\tilde{r}_1, \tilde{r}_2) = E[(\tilde{r}_1 - E(\tilde{r}_1))(\tilde{r}_2 - E(\tilde{r}_2))] = \sigma_{12}$$

$$\rho_{12} = \frac{Cov(\tilde{r}_1, \tilde{r}_2)}{\sigma_1 \sigma_2}$$

$$Var(\bar{R}_p) = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}$$

$$\beta_i = \frac{\text{Cov}(\tilde{r}_i, \tilde{R}_T)}{\text{Var}(\tilde{R}_T)}$$

$$\tilde{r}_i = \alpha_i + \beta_{i1}\tilde{F}_1 + \beta_{i2}\tilde{F}_2 + \cdots + \beta_{iK}\tilde{F}_K + \tilde{\varepsilon}_i, \text{ Factor Model}$$

$$\tilde{r}_i = r_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \cdots + \beta_{iK}\lambda_K + \tilde{\varepsilon}_i, \text{ APT Model}$$

## Derivatives

$$F_0 = S_0(1 + r_f)^T$$

$$f = (F_0 - K)(1 + r_f)^{-T}$$

$$c_0 - p_0 = S_0 - PV(K)$$

$$S_0 - K \leq C_0 - P_0 \leq S_0 - PV(K)$$

$$u = e^{\sigma\sqrt{T/N}}$$

$$d = \frac{1}{u}$$

$$\pi = \frac{1 + r_f - d}{u - d}$$

$$c_0 = S_0N(d_1) - PV(K)N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

## Real Investments:

$$E = P \times n$$

$$EV = D + E$$

$$NI = EBT(1 - T_c)$$

$$CE(\tilde{C}) = E(\tilde{C}) - b(\tilde{R}_T - r_f)$$

**Cumulative Normal Distribution**

<i>d</i>	<i>N(d)</i>	<i>d</i>	<i>N(d)</i>	<i>d</i>	<i>N(d)</i>	<i>d</i>	<i>N(d)</i>	<i>d</i>	<i>N(d)</i>	<i>d</i>	<i>N(d)</i>
-3,00	0,0013	-1,58	0,0571	-0,76	0,2236	0,06	0,5239	0,86	0,8051	1,66	0,9515
-2,95	0,0016	-1,56	0,0594	-0,74	0,2296	0,08	0,5319	0,88	0,8106	1,68	0,9535
-2,90	0,0019	-1,54	0,0618	-0,72	0,2358	0,10	0,5398	0,90	0,8159	1,70	0,9554
-2,85	0,0022	-1,52	0,0643	-0,70	0,2420	0,12	0,5478	0,92	0,8212	1,72	0,9573
-2,80	0,0026	-1,50	0,0668	-0,68	0,2483	0,14	0,5557	0,94	0,8264	1,74	0,9591
-2,75	0,0030	-1,48	0,0694	-0,66	0,2546	0,16	0,5636	0,96	0,8315	1,76	0,9608
-2,70	0,0035	-1,46	0,0721	-0,64	0,2611	0,18	0,5714	0,98	0,8365	1,78	0,9625
-2,65	0,0040	-1,44	0,0749	-0,62	0,2676	0,20	0,5793	1,00	0,8413	1,80	0,9641
-2,60	0,0047	-1,42	0,0778	-0,60	0,2743	0,22	0,5871	1,02	0,8461	1,82	0,9656
-2,55	0,0054	-1,40	0,0808	-0,58	0,2810	0,24	0,5948	1,04	0,8508	1,84	0,9671
-2,50	0,0062	-1,38	0,0838	-0,56	0,2877	0,26	0,6026	1,06	0,8554	1,86	0,9686
-2,45	0,0071	-1,36	0,0869	-0,54	0,2946	0,28	0,6103	1,08	0,8599	1,88	0,9699
-2,40	0,0082	-1,34	0,0901	-0,52	0,3015	0,30	0,6179	1,10	0,8643	1,90	0,9713
-2,35	0,0094	-1,32	0,0934	-0,50	0,3085	0,32	0,6255	1,12	0,8686	1,92	0,9726
-2,30	0,0107	-1,30	0,0968	-0,48	0,3156	0,34	0,6331	1,14	0,8729	1,94	0,9738
-2,25	0,0122	-1,28	0,1003	-0,46	0,3228	0,36	0,6406	1,16	0,8770	1,96	0,9750
-2,20	0,0139	-1,26	0,1038	-0,44	0,3300	0,38	0,6480	1,18	0,8810	1,98	0,9761
-2,15	0,0158	-1,24	0,1075	-0,42	0,3372	0,40	0,6554	1,20	0,8849	2,00	0,9772
-2,10	0,0179	-1,22	0,1112	-0,40	0,3446	0,42	0,6628	1,22	0,8888	2,05	0,9798
-2,05	0,0202	-1,20	0,1151	-0,38	0,3520	0,44	0,6700	1,24	0,8925	2,10	0,9821
-2,00	0,0228	-1,18	0,1190	-0,36	0,3594	0,46	0,6772	1,26	0,8962	2,15	0,9842
-1,98	0,0239	-1,16	0,1230	-0,34	0,3669	0,48	0,6844	1,28	0,8997	2,20	0,9861
-1,96	0,0250	-1,14	0,1271	-0,32	0,3745	0,50	0,6915	1,30	0,9032	2,25	0,9878
-1,94	0,0262	-1,12	0,1314	-0,30	0,3821	0,52	0,6985	1,32	0,9066	2,30	0,9893
-1,92	0,0274	-1,10	0,1357	-0,28	0,3897	0,54	0,7054	1,34	0,9099	2,35	0,9906
-1,90	0,0287	-1,08	0,1401	-0,26	0,3974	0,56	0,7123	1,36	0,9131	2,40	0,9918
-1,88	0,0301	-1,06	0,1446	-0,24	0,4052	0,58	0,7190	1,38	0,9162	2,45	0,9929
-1,86	0,0314	-1,04	0,1492	-0,22	0,4129	0,60	0,7257	1,40	0,9192	2,50	0,9938
-1,84	0,0329	-1,02	0,1539	-0,20	0,4207	0,62	0,7324	1,42	0,9222	2,55	0,9946
-1,82	0,0344	-1,00	0,1587	-0,18	0,4286	0,64	0,7389	1,44	0,9251	2,60	0,9953
-1,80	0,0359	-0,98	0,1635	-0,16	0,4364	0,66	0,7454	1,46	0,9279	2,65	0,9960
-1,78	0,0375	-0,96	0,1685	-0,14	0,4443	0,68	0,7517	1,48	0,9306	2,70	0,9965
-1,76	0,0392	-0,94	0,1736	-0,12	0,4522	0,70	0,7580	1,50	0,9332	2,75	0,9970
-1,74	0,0409	-0,92	0,1788	-0,10	0,4602	0,72	0,7642	1,52	0,9357	2,80	0,9974
-1,72	0,0427	-0,90	0,1841	-0,08	0,4681	0,74	0,7704	1,54	0,9382	2,85	0,9978
-1,70	0,0446	-0,88	0,1894	-0,06	0,4761	0,76	0,7764	1,56	0,9406	2,90	0,9981
-1,68	0,0465	-0,86	0,1949	-0,04	0,4840	0,78	0,7823	1,58	0,9429	2,95	0,9984
-1,66	0,0485	-0,84	0,2005	-0,02	0,4920	0,80	0,7881	1,60	0,9452	3,00	0,9987
-1,64	0,0505	-0,82	0,2061	0,00	0,5000	0,82	0,7939	1,62	0,9474	3,05	0,9989
-1,62	0,0526	-0,80	0,2119	0,02	0,5080	0,84	0,7995	1,64	0,9495		
-1,60	0,0548	-0,78	0,2177	0,04	0,5160						